Short Papers

Spectral-Domain Approach for Continuous Spectrum of Slot-Like Transmission Lines

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Abstract—**For the first time, the continuous spectrum part of slot-like transmission lines is described using the spectral-domain approach which has been successfully applied to the discrete part. Reliability of the approach is checked by numerical calculation of the surface current distribution across the slot plane in a simple illustrative example.**

I. Introduction

It is well known that surface waves do not form a complete set for open waveguides since the radiated field cannot be described by these modes alone [1]. Knowledge of the complete spectrum is required in order to analyze rigorously open discontinuities in which radiation cannot be neglected [2]. The discrete spectrum of slot-like lines (SL lines) has been successfully analyzed by the spectral-domain approach (SDA) [3]. It is the purpose of this paper to show that this technique also gives good results for the continuous spectrum.

II. Formulation

The SL lines under analysis (Fig. 1) consist of a combination of slots in an infinite conducting plane with a number of lossless dielectric layers superimposed on both sides. The $e^{-j\beta z}$ dependence and $e^{j\omega t}$ time variation are omitted in the analysis.

Using the Fourier transform in each region i ($i = O \cdots N$), the spectral densities of the axial field components of a continuous mode can be written as a combination of spectral plane waves

$$\begin{Bmatrix} \tilde{E}_{zi}(\alpha, y) \\ \tilde{H}_{zi}(\alpha, y) \end{Bmatrix} = \begin{Bmatrix} A_i(\alpha) \\ A'_i(\alpha) \end{Bmatrix} e^{-j\gamma_i y} + \begin{Bmatrix} B_i(\alpha) \\ B'_i(\alpha) \end{Bmatrix} e^{j\gamma_i y} \qquad (1)$$

where

$$\gamma_i^2 = \rho_i^2 - \alpha^2 \quad \rho_i^2 = k_i^2 - \beta^2 \quad k_i^2 = \frac{\omega^2}{c^2}\epsilon_i.$$

The phase constant β of the forward-traveling wave may be either real ($0 \leqslant \beta \leqslant k_0$) for propagating modes or imaginary ($-j\infty < \beta \leqslant j0$) for evanescent modes. The whole continuous spectrum is then obtained by summing the modal fields (1) over the above-mentioned ranges of β.

In both the discrete and continuous spectra, the condition to be imposed at infinity is that modal fields are bounded [1]. This condition uniquely defines the modal fields in regions O and N as the inverse Fourier transform of (1) requiring for every constituent plane wave

$$\operatorname{Im}\gamma_O = \operatorname{Im}\gamma_N \geqslant 0. \qquad (2)$$

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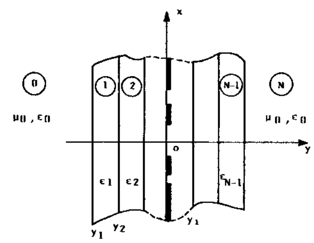

Fig. 1. Cross section of slot-like lines ("symmetric" configuration).

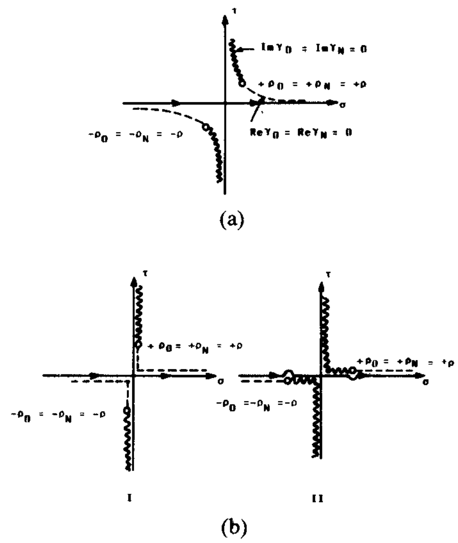

Fig. 2. Integration paths in the "proper sheet" in the α complex plane. (a) Lossy regions O and N. (b) Lossless regions O and N. I: Discrete spectrum case ($\rho^2 < 0$). II: Continuous spectrum case ($\rho^2 > 0$).

By assuming regions O and N to be lossy, the wiggly lines in Fig. 2(a) define the branch cuts of the double-valued functions $\gamma_O = \gamma_N$ in the complex $\alpha = \sigma + i\tau$ plane. The integration path from $\alpha = -\infty$ to $+\infty$ in Fig. 2(a) thus can be chosen as the real axis in the "proper" sheet of the α-plane where (2) is satisfied. Now, removing the losses in regions O and N, Fig. 2(b) describes integration paths for both the discrete spectrum with $\rho_O^2 = \rho_N^2 = \rho^2 \leqslant 0$ and the continuous spectrum with $\rho_O^2 = \rho_N^2 = \rho^2 > 0$. It can be noted in Fig. 2(b) that the discrete spectrum can use only the invisible range ($\gamma_O = \gamma_N = j\gamma$, $\gamma > 0$) of the plane-wave representation (1) for every real value of α lying on the integration path ($0 \leqslant |\alpha| < \infty$). So, in the whole spectral domain, we can write $B_O(\alpha) = B'_O(\alpha) = 0$ and $A_N(\alpha) = A'_N(\alpha) = 0$ in accordance with [3]. As for the plane-wave representation of the continuous spectrum, both invisible ($\gamma_O = \gamma_N = j\gamma$, $\gamma > 0$) and visible ($\gamma_O = \gamma_N = \gamma$, $\gamma > 0$) ranges must be used. Invisible and visible ranges correspond to parts $\rho < |\alpha| < \infty$ and $0 \leqslant |\alpha| < \rho$ of the spectral domain, respectively. The former is the evanescent part of the continuous spectrum for which we still have $B_O(\alpha) = B'_O(\alpha) = 0$ and $A_N(\alpha) = A'_N(\alpha) = 0$ and is related to the near-zone field. The latter is the propagating type and is responsible to the far-zone field; it provides the infinite complex power flow of a

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continuous mode [4] that can be written as

$$P = \delta(\gamma - \gamma')\left\{ \frac{\omega\epsilon_0\beta}{4\rho^2}\int_0^{\rho}\left(|A_O(\alpha)|^2 + |B_O(\alpha)|^2 + |A_N(\alpha)|^2 + |B_N(\alpha)|^2\right) d\alpha + \frac{\omega\mu_0\beta^*}{4\rho^2}\int_0^{\rho}\left(|A'_O(\alpha)|^2 + |B'_O(\alpha)|^2 + |A'_N(\alpha)|^2 + |B'_N(\alpha)|^2\right) d\alpha \right\}. \qquad (3)$$

From (3), it can be seen that a power separation arises between constituent spectral plane waves of TE and TM types as well as between spectral plane waves of a given type radiating in either the y or $-y$ direction. Therefore, continuous field solutions have to be constructed from four partial scattered fields corresponding to the illumination of the SL lines by TE and/or TM incident spectral plane waves denoted $A'_O(\alpha)e^{-j\gamma y}$ and/or $A_O(\alpha)e^{-j\gamma y}$, respectively, in region O, and TE and/or TM incident spectral plane waves denoted $B'_N(\alpha)e^{j\gamma y}$ and/or $B_N(\alpha)e^{j\gamma y}$, respectively, in region N.

These incident waves with arbitrary amplitudes and phases are created by filamentary sources at infinity. Selecting, for instance, the TE incident spectral wave $A'_O(\alpha)e^{-j\gamma y}$, we must write $A_O(\alpha) = 0$ and $B_N(\alpha) = B'_N(\alpha) = 0$ in the visible range of (1). Let us notice that the invisible range of the spectral domain does not exist in "symmetric" multilayered waveguides ($\epsilon_O = \epsilon_N$) with homogeneous boundaries at the interface $y = 0$ [4]. On the contrary, this range is used in an "asymmetric" configuration ($\epsilon_O \neq \epsilon_N$) [5]. For each partial field, both homogeneous and inhomogeneous boundary conditions at interfaces y_i can be written in a general matrix notation [6]. This leads to pairs of functional equations relating the spectral densities of the tangential electric field to those of the surface current at the slot plane $y = 0$. They are written as

$$[G^{\text{vis}}(\alpha,\beta)] \cdot \begin{bmatrix} \tilde{E}_x(\alpha,0) \\ \tilde{E}_z(\alpha,0) \end{bmatrix} = \begin{bmatrix} \tilde{J}_x(\alpha,0) \\ \tilde{J}_z(\alpha,0) \end{bmatrix} + \begin{bmatrix} \Delta_1(\alpha) \\ \Delta_2(\alpha) \end{bmatrix} \qquad (4a)$$

for the visible (vis) range of the spectral domain, and as

$$[G^{\text{inv}}(\alpha,\beta)] \cdot \begin{bmatrix} \tilde{E}_x(\alpha,0) \\ \tilde{E}_z(\alpha,0) \end{bmatrix} = \begin{bmatrix} \tilde{J}_x(\alpha,0) \\ \tilde{J}_z(\alpha,0) \end{bmatrix} \qquad (4b)$$

for the invisible (inv) range of the spectral domain. Quantities $\Delta_1(\alpha)$ and $\Delta_2(\alpha)$ in (4a), which are functions of the amplitude of the selected incident TE or TM spectral plane wave, represent sources at infinity. Obviously, no sources appear in (4b). Then, (4) can be solved in the spectral domain by using the Galerkin procedure as in [3]. Here, a set of inhomogeneous linear equations (deterministic problem) is obtained, whose solution gives the spectral densities of the partial hybrid field under consideration for each permissible value of the phase constant β.

III. Numerical Results

To verify the reliability of the method, the single-slot configuration without dielectric layers has been examined. Such an SL line supports continuous waves only. The partial field that corresponds to the TE spectral wave $A'(\alpha)e^{-j\gamma y}$ incident in the y direction becomes purely TE so that $E_z = H_z = 0$ anywhere. We have in the invisible range $B'_O(\alpha) = A'_1(\alpha) = 0$, while in the visible one, $B'_1(\alpha) = 0$ and $A'_O(\alpha) = 1$; such a source normalization in the spectral domain has to be distinguished from the field normalization given by (3).

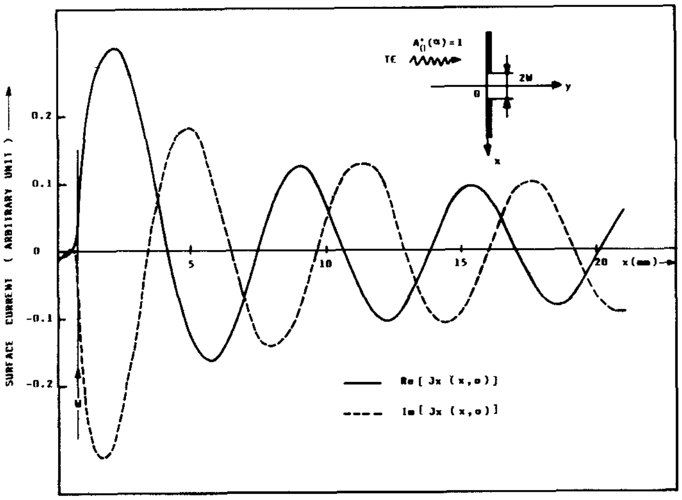

Fig. 3. Current density distribution $J_x(x,0)$ in the slot plane corresponding to a given partial field. $N_b = 2$, $F = 3$ GHz, $\beta = -j1$ rd/mm, $2W = 1.66$ mm.

The E_x electric-field component across the slot aperture ($|x| < W$) for all partial hybrid fields was expanded as

$$E_x(x,0) = a_0 \Big/ \sqrt{1 - \left(\frac{x}{W}\right)^2} + \sum_{n=1}^{N_b} a_n \cos\frac{n\pi}{W}x. \qquad (5)$$

Equations (4) have been solved numerically to yield the spectral partial field densities $\tilde{E}_x(\alpha,0)$ and $\tilde{J}_x(\alpha,0)$. The electric field $E_x(x,0)$ always satisfies the inhomogeneous boundary conditions at the slot plane $y = 0$ because of the choice of expanding functions (5). To confirm the validity of the method, the surface current $J_x(x,0)$ must verify the prescribed inhomogeneous boundary conditions at the slot plane

$$J_x(x,0) = 0, \quad |x| < W$$
$$J_x(x,0) \neq 0, \quad |x| \geq W. \qquad (6)$$

Fig. 3 shows real and imaginary parts of the surface current at the slot plane $y = 0$. Inasmuch as the current in the aperture is found insignificant in comparison with that on the conducting half plane, conditions (6) are satisfied.

IV. Conclusion

The spectral-domain approach for the continuous spectrum of slot-like lines is presented. Numerical results obtained for one of the four partial fields in a single-slot without dielectric layers confirm the reliability of the analysis. The method can be easily extended for microstrip-like transmission lines. Further results will be presented in the near future.

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